[**You may use one side of an 8.5 $\times$ 11 inch sheet of paper and a calculator.**] Show all of your work clearly in the space provided or on the additional page at the end of the exam. If the additional page is used, clearly identify to which exam question it is related. Be sure to **read each problem carefully**. You should answer all 4 questions. Note that the exam is double sided.

1. Suppose two points (x_0, y_0) and (x_1, y_1) are on a straight line with $y_1 \neq y_0$. The following two equations may be used to find the $x-$intercept of the line:

$$x_a = x_0 - \frac{(x_1 - x_0)y_0}{y_1 - y_0} \qquad \text{and} \qquad x_b = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0}$$

(a) (10 points) Show that both equations yield the $x-$intercept of the line segment.

(b) (10 points) Calculate the $x-$intercept for the line segment specified by $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$ (for both methods) using **three-significant figure** arithmetic.

(c) (10 points) Which equation is better suited for a numerical calculation of the $x-$intercept, and what characteristics about the equation make it better?

2. (15 points) Compare and contrast Gauss–Jacobi and Gauss–Seidel iteration techniques. What are the advantages and disadvantages of each?

3. (10 points) Find the $\infty-$norm of the following matrix:

$$\mathbf{A} = \begin{bmatrix} -\cos(30) & \sin(30) & 0 \\ \sin(30) & 0 & \cos(30) \\ 0 & 0 & 1 \end{bmatrix}$$

Be sure to show all your work.

4. Consider a new method for finding a root of a function which we will call the **modified Newton** method. The method is described by the update equation:

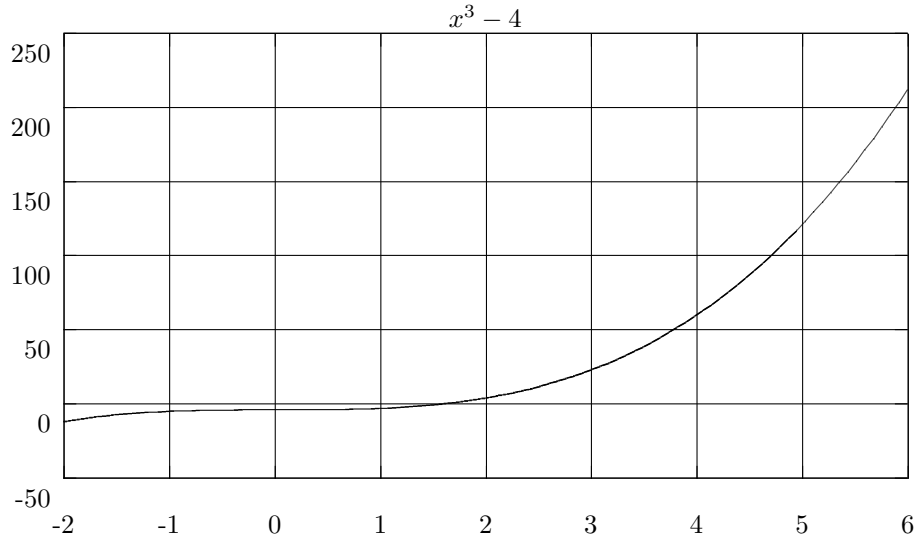
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_0)}$$

where $f'(\cdot)$ is the derivative of $f(\cdot)$ with respect to x.

(a) (10 points) How does the **modified Newton method** differ from **Newton's method**?

(b) (10 points) Treat the modified Newton method like any of the other iterative methods discussed in the text or lecture. For the given function $f(x) = x^3 - 4$ with $x_0 = 4$, what is the iterative function $g(x)$ for the modified Newton method?

(c) (15 points) Indicate the first three approximations (x_1, x_2, and x_3) of the root of $f(x) = x^3 - 4$ on the graph of $f(\cdot)$ below. Use the modified Newton method and an initial value of $x_0 = 4$. Be sure to show all of your work.



(d) (10 points) Generally speaking, which method (modified Newton or Newton's) do you think converges more rapidly? Be sure to explain your reasoning.

Additional work area for any problem. Clearly identify to which problem the work on this page is related.