[**Closed book/notes, you may use one double-sided 8.5 $\times$ 11 inch sheet of paper**] Show all of your work clearly in the space provided or on the additional page at the end of the exam. If the additional page is used, clearly identify to which exam question it is related. Be sure to **read each problem carefully**. You should answer all 5 questions, and you may wish to answer the bonus question if you have time. Note that the exam is double sided.

1. (10 points) Briefly, in your own words, explain the characteristics of a **stable** sorting algorithm.

2. (15 points) Consider the following algorithm:

```
SelectionSort(int n, array A[1..n]) {
  for i ← 1 to n−1 do {
    min ← i
    for j ← i+1 to n do {
      if( A[j] < A[min]) {
        min ← j
      }
    }
    swap A[i] with A[min]
  }
}
```

Give an example input array, A[1..n], for which the SelectionSort algorithm is unstable. (Full credit will be given for the smallest possible array size.) Be sure to explain your example.

3. This multi-part problem involves developing and analyzing an algorithm which takes an array of real numbers, A[1..n], and determines whether or not there exist two elements, A[i] and A[j] such that A[i] = -A[j]. (If A contains zero, the answer is yes.) To receive full credit, the algorithm you develop must have a worst case time complexity of $\Theta(n \log n)$.

(a) (15 points) Describe an algorithm that meets the above criteria. You may use C++, pseudocode, or unambiguous English prose to describe your algorithm.

(b) (10 points) Argue the time complexity of the algorithm you described in part **(a)**.

4. Consider the following recurrence:

$$T(n) = \begin{cases} 2 & \text{if } n = 1, \\ 1 & \text{if } n = 2, \\ T(n-2) + T(2) + n & \text{if } n > 2. \end{cases}$$

(a) (15 points) Solve the recurrence, and be sure to state any assumptions you make.

(b) (10 points) Is this result an upper-bound, lower-bound, or tight bound? State the time complexity using the appropriate notation.

5. Joan works at McDonald's and recently read a report indicating that many diseases are spread through casual contact with United States currency. She is concerned about excessive contact with money while making change.

(a) (15 points) Describe a greedy algorithm for her which will allow her to calculate the customer's correct change while minimizing the number of monetary items (bills and coins).

(b) (10 points) Suppose McDonald's develops a mail-order business and Joan is transferred to help put the appropriate postage on items that are ready to be shipped. The post office has 1¢, 2¢, 3¢, 4¢, 5¢, 10¢, 15¢, 20¢, 23¢, 32¢, 40¢, 45¢, 50¢, 60¢, 65¢, 78¢, \$1, \$2, \$3, \$5, and \$10.75 stamps available. Joan's greedy algorithm will not work for minimizing the total number of stamps needed to achieve the appropriate postage. Provide an example of a postage amount that would cause the greedy algorithm to produce a sub-optimal solution.

bonus (5 points) Suppose you are given an vector containing (in random order) all but one of the integers from 1 to n. Write a C++ function that will determine which integer is missing from the sequence. The algorithm should execute in $O(n)$ time. (No partial credit will be given for the bonus question.)

Additional work area for any problem. Clearly identify to which problem the work on this page is related.